Прикладна математика

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METHOD FOR REFINING WEIGHTS IN MULTI-CRITERIA UTILITY FUNCTION IN MAUT

Background. In modern multi-criteria decision-making, a critical challenge is the determination of weight coefficients in the utility function. Classical MAUT (Multi-Attribute Utility Theory) methods often rely on subjective expert evaluations, leading to potential errors due to expert fatigue and the limited number of comparisons. Additionally, discrepancies in the total weight sum can violate the axioms of linear convolution.
Objective. The paper aims to develop a method for refining weight coefficients in the multi-attribute utility function of MAUT, which reduces the influence of subjectivity and ensures analytically consistent values.
Methods. An approach based on the Lagrange method applied to a system of normalised weights is proposed. This method transforms relative (non-normalised) expert assessments into precise weights by solving a system of equations analytically. To minimise errors, only relative weight ratios are used, reducing the number of expert queries from quadratic to linear complexity.
Results. A formula for refining weight coefficients is derived, preserving relative expert evaluations while ensuring accuracy and normalisation. An example involving four criteria demonstrates the use of Lagrange multipliers to achieve refined weights with an error margin below 0.001. The method provides stable and analytically sound results without requiring complete pairwise comparisons.
Conclusions. The proposed method enables efficient refinement of weight coefficients in MAUT without overburdening experts. Analytical computation reduces error risks and enhances decision-making objectivity. The method is suitable for tasks with numerous criteria and offers a robust foundation for constructing utility functions in multi-criteria models.
Keywords: MAUT; utility function; weight coefficients; Lagrange method; expert evaluation; multi-criteria decision-making.

Problem Statement

In the modern world, where decisions often require considering numerous factors, Multi-Attribute Utility Theory (MAUT) has become a key tool for analysing complex simplex. This theory is an extension of classical utility theory, adapted to tasks where it is necessary to consider not one but multiple criteria simultaneously.

MAUT allows for the formalisation of the decision-making process by combining various aspects of choice into a singular analytical approach. It is based on the principles of rationality and assumes that the preferences of an individual or organisation can be expressed through a function that reflects the degree of satisfaction for each criterion.

The application of Multi-Attribute Utility Theory spans a wide range of fields: from strategic management and planning to environmental policy and engineering design. This article examines the theoretical foundations of MAUT, methods for its practical application, and its role in improving the quality of decisions in complex multi-criteria tasks. Multi-Attribute Utility Theory (MAUT) enables the following tasks to be addressed [1]:

– to construct a mathematically justified utility function;

– to verify certain conditions that determine the form of the function in dialogue with the decision-maker (DM);

– to rank all possible alternatives by quality and evaluate them based on the identified decision rule.

The MAUT method is best suited for tasks with a large number of alternatives. Main Stages of the MAUT Method:

Let us outline the stages of solving a problem using Multi-Attribute Utility Theory:

1. To develop a list of criteria.

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2. To construct a utility function for each criterion.

3. To verify the conditions that determine the general form of the utility function.

4. To establish a relationship between the evaluation of alternatives by criterion and the overall quality of alternatives.

5. Evaluate all available alternatives and select the best one.

According to classical utility theory, Multi-Attribute Utility Theory is based on axiomatic principles. The conditions that a utility function must satisfy are formulated as axioms. If a condition is met, it serves as proof of the existence of the utility function. In MAUT, these conditions can be divided into two groups:

1. General axioms, which are used in utility theory.

2. Independence axioms, specific to MAUT.

In this study, we focused on the axioms of the second group [2].

Main approaches to verifying criteria independence and refining weight coefficients in MAUT

Let us present several independence conditions that belong to the second group of axioms.

1. Difference Independence: Preferences between two alternatives that differ only in their evaluations on an ordinal scale for one criterion C_1 do not depend on the identical evaluations for other criteria $C_2,...,C_t$.

2. Utility Independence: A criterion C_1 is said to be utility independent of criteria $C_2,...,C_t$ if the preference order of lotteries, in which only the levels of criterion C_1 vary, does not depend on the fixed values of the other criteria.

3. Preferential Independence: Two criteria C_1 and C_2 are preferentially independent of the other criteria $C_3,...,C_t$ if the preferences between alternatives that differ only in their evaluations of C_1 and C_2 do not depend on the fixed values of the other criteria.

The first two independence conditions pertain to the independence of one criterion from others, while the third condition pertains to the independence of several criteria from others.

Main Theorem: If the axioms of the first group and some independence conditions are satisfied, then it strictly follows that a multi-criteria utility function exists in a specific form.

We can formulate R. Keeney's theorem [3], which underlies practical methods for evaluating alternatives: If the conditions of utility independence and preferential independence are satisfied, then the utility function is additive:

$$U(x) = \sum_{i=1}^{N} w_i U_i(x),$$

with $\sum_{(i=1)}^{N} w_i = 1$, or multiplicative:

$$1 + kU(x) = \prod_i (1 + kw_i U_i)$$

with $\sum_{(i=1)}^{N} w_i \neq 1$, where

– Let U and U_i be utility functions ranging from 0 to 1;

– w_i be the coefficients (weights) of the criteria, where $0 < w_i < 1$;

– and k be a coefficient such that $k > -1$.

Thus, the multi-criteria utility function can be defined if the values of the coefficients w_i and the single-criterion utility functions $U_i(x)$ are known.

Knowing the range of evaluations for each criterion, we construct a function that determines the utility for experts of each evaluation within this range. The maximum value of this function is set to one, and the minimum value to zero. To determine intermediate values, deterministic lotteries are used, depending on the specific task. Examples of their construction are presented in [2].

To determine the overall utility function, it is necessary to verify the conditions of utility independence and preferential independence. The verification of utility independence can be combined with the preliminary stage of constructing single-criterion utility functions.

First, the expert is informed that when determining equivalent values for certainty, they should consider that the other criteria have better values. Then, the expert is presented with the same task but assumes that the other alternative has the worst value (similar to the procedure for verifying utility independence [2, 4]). If the certainty equivalent is the same in both cases, it can be concluded that the given criterion is utility independent of the other criteria.

Note that to fully verify the utility independence condition, this check should be performed for all lotteries. However, it is usually sufficient to perform an approximate check using the first lottery, which is used only during the construction of the single-criterion utility function.

During the verification of the independence condition, the main consideration is to draw a plane along the axis of two evaluated criterion values.

For a complete verification of the preferential independence condition, all pairs of criteria should be considered. However, during an approximate check, one or two of the most important criteria are selected, and the other criteria are considered only in combination with them [2, 5].

Strictly speaking, intermediate values should also be taken into account, but in general, such a check is considered sufficient [2, 6].

MAUT relies largely on the concept of weights (importance coefficients) for criteria. It is assumed that experts can determine the coefficients – numbers that reflect the importance of a criterion. The relationship between the weights of the criteria is established by identifying indifference points on the planes of two criteria. Unlike testing preferences for independence conditions, the axis ranks the criterion values from worst to best.

The main premise on which the classical methods of Keeney [7], Raiffa [3], and Fishburn [8] are based (explicitly or implicitly) is that experts are not mistaken when providing estimates, and the desired utility function should correspond as closely as possible to their estimates. For this purpose, an additive form of the utility function was developed, which is a direct reflection of the independence of the criteria for the case when the sum of the weights is equal to one, as well as a multiplicative form, which is essentially additive, with the counterintuitive occurrence of weight coefficients in the case when their sum is not equal to one. That is, the second case is considered a variant of the norm. This approach solves the problem of constructing the utility function, but it is difficult to consider it consistent. Because the sum of the weight coefficients by definition is equal to one, which is also the basis of the well-known metamodel of multi-criteria decisions – the linear convolution method. It is on this understanding that the intuition of weight coefficients, including among experts, is based. Then the fact of obtaining estimates of weight coefficients whose sum is not equal to one is not fundamental in nature, but is a common consequence of the existence of errors in expert assessment. Usually, experts answer a large number of thematic questions, which leads to their overload and fatigue, and as a result to errors in assessments. Not to mention the fact that infallibility is not inherent in human nature at all, even in the nature of experienced specialists. In this study, we adopt the stance that the multi-criteria utility function, when the criteria are independent, invariably takes an additive form. In this case, it is not necessary to require experts that the sum of weight coefficients be unity. It is also not necessary to conduct a full set of pairwise comparisons, which will reduce the load on experts to a minimum sufficient: the number of calls drops from $\frac{(n-1)^2}{2}$ to n, which reduces the number of errors both in general and due to fatigue:

$$\begin{cases} \sum_{i=1}^{n} w_i = 1; \\ \sum_{i=1}^{n} w_i U_i = U. \end{cases}$$

When a problem of adjusting coefficients was solved, we have the opportunity to more accurately and analytically identify the criteria that are important to us and eliminate the subjectivity of experts. The approach described below allows us to obtain refined weight estimates using the Lagrange method and a certain formalised collection of primary information from experts.

Method for refinement of weight estimates for multi-criterion utility function in MAUT

In most cases, when evaluating criteria, an expert can provide a relative assessment of the impact of one alternative more confidently compared to another rather than an absolute one. Therefore, when forming a multi-criteria utility function, we focus on the relationship between the weights (influence) of each utility function. By default, we assume that the expert provides non-normalised estimates [2, 3, 9], meaning the sum of the weights does not equal one. However, these estimates can be adjusted to normalised values through a normalisation procedure. Although Keeney's theorem allows us to describe the utility function as multiplicative, this introduces more degrees of freedom and, consequently, accumulates errors in various types of evaluations. Therefore, it is desirable to obtain more accurate estimates derived analytically but based on real facts.

Suppose we do not know the exact estimates w_i, but the expert can provide their subjective estimates of the ratio of weights between criteria, i.e., $\frac{w_i}{w_j} \approx a_{ij}$, but $\prod a_{ij} \neq 1$. Our task is to standardise these estimates so that the product equals 1.

Let us present the procedure for transitioning from non-normalised estimates to normalised ones [11]. Suppose our weights are as follows:

$$\begin{cases} \dfrac{w_i}{w_j} \approx \alpha_{ij}; \\ \prod_{ij} \alpha_{ij} \neq 1; \\ \sum_{i=1}^{n} w_i \neq 1. \end{cases} \quad (1)$$

To switch from multiplicative form, we need to redefine them like this:

$$\begin{cases} \dfrac{w_i}{w_j} = \beta_{ij}; \\ \prod_{ij} \beta_{ij} = 1; \\ \sum_{i=1}^{n} w_i = 1. \end{cases} \quad (2)$$

To find them, we use the least squares method:

$$\begin{cases} \sum_{i=1}^{n} w_i = 1; \\ 0 < w_i < 1; \\ \sum (\dfrac{w_i}{w_j} - \alpha_{ij})^2 \to \min. \end{cases}$$

To solve this problem, we use the Lagrange method [10]:

$$L = \sum (\frac{w_i}{w_j} - \alpha_{ij})^2 + \lambda(\sum_{i=1}^{n} w_i - 1)$$

where L – Lagrange function.

Next, we take the derivatives for each w_i:

$$\frac{\partial L}{\partial w_i} = \sum \frac{\partial L_k}{\partial w_i} + \lambda,$$

where L_k – one of the appendices $(\frac{w_i}{w_j} - \alpha_{ij})^2$.

To further simplify, multiply by the corresponding variable for which we took the derivative:

$$w_i \frac{\partial L}{\partial w_i} = 2\sum \frac{w_i}{w_j}(\frac{w_i}{w_j} - \alpha_{ij}) + \lambda w_i,$$

where the sum is carried out over the remaining terms of the derivative, and the two appears from the square when taking the derivative.

The general system of partial derivatives will look like this $(\beta_{ij} = \frac{w_i}{w_j})$:

$$\begin{cases} \sum_{i=1}^{n} w_i = 1; \\ 0 < w_i < 1; \\ \sum \beta_{ij}(\beta_{ij} - \alpha_{ij}) = \dfrac{\lambda w_i}{2}. \end{cases}$$

If we sum all the transformed partial derivatives, it turns out that on the left side, all terms cancel each other out due to the alternating signs (pluses and minuses) in the different partial derivatives. On the right side, the sum of the weights appears:

$$\Delta I_i = I_{(-i)} - I = (1 - M_{(-i)}) - \\ - (1 - M) = M - M_{(-i)}.$$

From this, it follows that all terms are equal to each other:

$$\beta_{12}(\beta_{12} - \alpha_{12}) = \ldots = \beta_{ij}(\beta_{ij} - \alpha_{ij}) = \ldots = \beta_{mn}(\beta_{mn} - \alpha_{mn}).$$

Clarification. The indices must be cyclic, not equal to each other, and not symmetric, while covering all possible combinations of the available weights.

From the last equation, we can formulate a parametric quadratic equation for each

$$\beta_{ij}(\beta_{ij} - \alpha_{ij}) = \mu;$$
$$\beta_{ij}^2 - \beta_{ij}\alpha_{ij} - \mu = 0;$$
$$D = \sqrt{\alpha_{ij}{}^2 + 4\mu};$$
$$\beta_{ij}^{1,2} = \frac{1}{2}(\sqrt{\alpha_{ij}^2 + 4\mu}).$$

Two cases are considered:

A. $\prod \alpha_{ij} > 1, \beta_{ij} < \alpha_{ij} \Rightarrow \mu < 0$.

B. $\prod \alpha_{ij} < 1, \beta_{ij} > \alpha_{ij} \Rightarrow \mu > 0$.

In both cases, to solve the quadratic equation, we choose the option with a plus sign, because it will be much closer to a_{ij} than the option with a minus, and by formulating the problem, we try to adjust the available estimates analytically, rather than strictly correcting them.

Than:

$$\prod \beta_{ij} = 1 \Rightarrow \prod (\alpha_{ij} + \sqrt{\alpha_{ij}{}^2 + 4\mu}) = 2^n, \quad (3)$$

where n – number of factors. The two is obtained from the roots of the quadratic equation with respect to β_{ij}. Knowing β_{ij} we can express w_i:

$$w_i = \frac{1}{1+\beta_{ij}+\beta_{jk}\beta_{kl}+\ldots+\beta_{jk}\beta_{kl}\ldots\beta_{vz}},$$

where $i, j, \ldots, z$ are all combinations of indices according to the number of weight coefficients.

So, with known a_{ij}, the previous equation, although it turns out to be irrational, is an equation with one variable and can be solved using numerical methods on a computer.

Let's give a small example. Suppose we need to build a multi-criteria utility function consisting of four criteria. Experts can provide us with relative estimates of the ratios for these criteria that satisfy conditions (1).

So it lets:

$$\alpha_{12}=0,5;\alpha_{23}=2;\alpha_{34}=3;\alpha_{41}=0,4.$$

Then it means:

$$\prod \alpha_{ij}=1,2.$$

This indicator is close to unity, but not equal to it, so it is necessary to run the procedure for correcting the weight estimates to use the additive loss function in the future.

Let's write a condition for finding normalized estimates:

$$\begin{cases}\sum_{i-=1}^{4} w_i=1;\\ 0<w_i<1;\\ (\beta_{12}-\alpha_{12})^2+(\beta_{23}-\alpha_{23})^2+(\beta_{34}-\alpha_{34})^2+(\beta_{41}-\alpha_{41})^2\to\min.\end{cases}$$

Let's write the Lagrange equation and substitute a_{ij}:

$$L=(\beta_{12}-0,5)^2+(\beta_{23}-2)^2+(\beta_{34}-3)^2+(\beta_{41}-0,4)^2-\\-(w_1+w_2+w_3+w_4).$$

Next, we take the partial derivatives and set them equal to zero:

$$\begin{cases}\frac{\partial L}{\partial w_1}=\frac{2}{w_2}(\beta_{12}-0,5)-\frac{2w_4}{w_1^2}(\beta_{41}-0,4)-\lambda=0;\\ \frac{\partial L}{\partial w_2}=\frac{2}{w_3}(\beta_{23}-2)-\frac{2w_1}{w_2^2}(\beta_{12}-0,5)-\lambda=0;\\ \frac{\partial L}{\partial w_3}=\frac{2}{w_4}(\beta_{34}-3)-\frac{2w_2}{w_3^2}(\beta_{23}-2)-\lambda=0;\\ \frac{\partial L}{\partial w_4}=\frac{2}{w_1}(\beta_{41}-0,4)-\frac{2w_3}{w_4^2}(\beta_{34}-3)-\lambda=0.\end{cases}$$

Multiply each equation by the corresponding w_i:

$$\begin{cases}\frac{\partial L}{\partial w_1}\cdot w_1=\frac{2w_1}{w_2}(\beta_{12}-0,5)-\frac{2w_4w_1}{w_1^2}(\beta_{41}-0,4)-\lambda w_1=0;\\ \frac{\partial L}{\partial w_2}\cdot w_2=\frac{2w_2}{w_3}(\beta_{23}-2)-\frac{2w_1w_2}{w_2^2}(\beta_{12}-0,5)-\lambda w_2=0;\\ \frac{\partial L}{\partial w_3}\cdot w_3=\frac{2w_3}{w_4}(\beta_{34}-3)-\frac{2w_2w_3}{w_3^2}(\beta_{23}-2)-\lambda w_3=0;\\ \frac{\partial L}{\partial w_4}\cdot w_4=\frac{2w_4}{w_1}(\beta_{41}-0,4)-\frac{2w_3w_4}{w_4^2}(\beta_{34}-3)-\lambda w_4=0.\end{cases}$$

We sum the equations and get:

$$-\frac{\lambda}{2}\sum_{i=1}^{n} w_i=0.$$

Which was to be proved.

So, we can use the results obtained in the general case (3):

$$\prod(\alpha_{ij}+\sqrt{\alpha_{ij}^{\ 2}+4\mu})=2^n;$$

$$(0,5+\sqrt{0,5^2+4\mu})(2+\sqrt{2^2+4\mu})(3+\sqrt{3^2+4\mu})\times\\ \times(0,4+\sqrt{0,4^2+4\mu})=16.$$

Using numerical methods and with the help of a computer, we obtain approximate values of μ. Of all the values found, $\mu = -0.015$ suits us, because the others have positive values, and according to condition (2), we need negative values.

So, the values of β_{ij} have the following values:

$$\beta_{12}=\frac{0,5+\sqrt{0,5^2+4\mu}}{2};\beta_{23}=\frac{2+\sqrt{2^2+4\mu}}{2};$$

$$\beta_{34}=\frac{3+\sqrt{3^2+4\mu}}{2};\beta_{41}=\frac{0,4+\sqrt{0,4^2+4\mu}}{2}\Rightarrow$$

$$\Rightarrow\beta_{12}=0,53;\beta_{23}=2,01;\beta_{34}=3,005;\beta_{41}=0,43.$$

Next, from the system of equations we find the normalized values of w_i:

$$\begin{cases}\sum_{i=1}^{4} w_i=1;\\ w_1=0,53w_2;\\ w_2=2,01w_3;\\ w_3=3,005w_4;\\ w_4=0,43w_1.\end{cases}$$

We solve the system and get:

$$w_1=0,217;w_2=0,409;w_3=0,093;w_1=0,2805.$$

The weights are rounded to the third sign, since there are irrational solutions.

Based on the obtained weights, we have the general form of the loss function for these estimates:

$$U = 0,217U_1 + 0,409U_2 + 0,093U_3 + 0,2805U_4.$$

Further, based on the utility functions for each alternative, we can obtain an analytical estimate of the total utility from this combination of utility functions.

Conclusions

Existing methods in the field of multi-attribute utility theory (MAUT) often rely on subjective expert judgments and complete pairwise comparisons to determine weight coefficients, which results in increased cognitive load and a high risk of inconsistency. While previous research has proposed heuristic or numerical approaches to mitigate expert errors, these methods typically lack mathematical rigor and offer limited analytical transparency.

This study presents a novel analytical solution based on the Lagrange method for refining weight coefficients in MAUT. Unlike conventional techniques, the proposed method maintains the relative importance assigned by experts while transforming it into a normalized and consistent form. A significant advantage of this approach lies in its reduction of the required number of expert inputs from quadratic to linear scale, which minimizes cognitive fatigue and enhances the practicality of expert-based decision models. Moreover, the method produces mathematically grounded results with high stability and repeatability, making it suitable for applications in strategic planning, public administration, technical systems design, and other domains involving complex multi-criteria evaluations.

The proposed technique opens the door to further developments, such as the integration of fuzzy or linguistic inputs and the extension to nonlinear or group-based decision-making models. Future research could focus on the incorporation of probabilistic or Bayesian mechanisms for collective expert input refinement, as well as on the development of software tools to support practical implementation in decision support systems. These enhancements would further expand the applicability and robustness of the method in real-world environments characterized by uncertainty and complexity.

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МЕТОД УТОЧНЕННЯ ВАГ У БАГАТОКРИТЕРІАЛЬНІЙ ФУНКЦІЇ КОРИСНОСТІ В MAUT

Проблематика. У задачах прийняття рішень (ПР) критичним моментом є визначення вагових коефіцієнтів у багатокритеріальній функції корисності. Класичні методи MAUT (багатокритеріальної теорії корисності) спираються на суб'єктивні оцінки експертів і не є стійкими до помилок через їх перевантаження і втому. Крім того, відхилення суми ваг надмірно впливає на значення й вигляд функції корисності.

Мета дослідження. Розробити метод уточнення оцінок значень вагових коефіцієнтів у багатокритеріальній функції корисності MAUT, що забезпечує їх аналітичну узгодженість і зменшує вплив помилок експертів.

Методика реалізації. Запропоновано підхід, оснований на МНК, застосовний до системи нормалізованих ваг. Цей метод перетворює експертні оцінки відношень ваг на нормалізовані ваги шляхом аналітичного розв'язання системи рівнянь. Для зменшення помилок використано мінімальний набір відношень ваг, що мінімізує потрібну кількість експертних оцінок.

Результати дослідження. Формула для вагових коефіцієнтів на основі експертних оцінок їх відношень забезпечує найкращу точність і нормалізацію. Приклад із чотирма критеріями використовує метод множників Лагранжа для знаходження ваг із похибкою менше 0,001. Метод забезпечує стабільні та аналітично обґрунтовані результати за мінімальної кількості попарних порівнянь.

Висновки. Запропонований метод дозволяє ефективно знаходити вагові коефіцієнти MAUT з мінімальним навантаженням експертів. Результати зменшують вплив суб'єктивних помилок і підвищують якість прийняття рішень. Метод підходить для задач ПР з кількісними критеріями і пропонує надійну основу для побудови узагальненого критерію.

Ключові слова: MAUT; функція корисності; вагові коефіцієнти; метод Лагранжа; експертне оцінювання; багатокритеріальне прийняття рішень.

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